Kinematics & Dynamics of Linkages

Lecture 13: 3 Positions Analytical Synthesis

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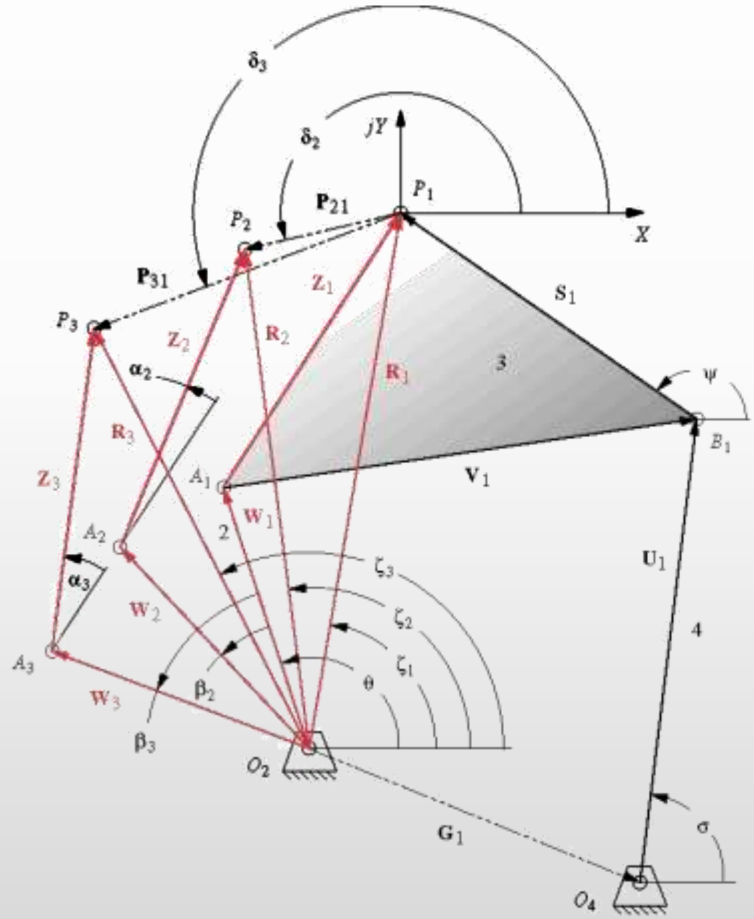
LAU

3-Position Motion Generation with Fixed Pivots

Design a four bar linkage that will move a line on its coupler link such that a point P on that line will be first at P1 , P2 and later P3 and will Also rotate the line through an angle α_2 and then α_3 assuming fixed pivots O_2 and O_4



1st Dyade: WZ

Define the following variables:

$\boldsymbol{w}$ = length **link2**

$\boldsymbol{\theta}$ = initial angle of **link2**

$\boldsymbol{\beta_2}$ = 1st change in angle of **link2**

$\boldsymbol{\beta_3}$ = 2nd change in angle of **link2**

$\boldsymbol{z}$ = distance from $\boldsymbol{A}$ and $\boldsymbol{P}$

$\boldsymbol{\Phi}$ = initial angle of **link** $\boldsymbol{z}$

$\boldsymbol{\alpha_2}$ = 1st change in angle of **link** $\boldsymbol{z}$

$\boldsymbol{\alpha_3}$ = 2nd change in angle of **link** $\boldsymbol{z}$

$\boldsymbol{p}_{21}$ = distance from point $\boldsymbol{P_1}$ to $\boldsymbol{P_2}$

$\boldsymbol{p}_{31}$ = distance from point $\boldsymbol{P_1}$ to $\boldsymbol{P_3}$

$\boldsymbol{\delta_2}$ = orientation angle of line from $\boldsymbol{P_1}$ to $\boldsymbol{P_2}$

$\boldsymbol{\delta_3}$ = orientation angle of line from $\boldsymbol{P_1}$ to $\boldsymbol{P_3}$

$\boldsymbol{R_1}$ = position vector $\mathbf{O_2P_1}$

$\boldsymbol{R_2}$ = position vector $\mathbf{O_2P_2}$

$\boldsymbol{R_3}$ = position vector $\mathbf{O_2P_3}$

$\boldsymbol{\xi_1}$ = orientation angle of $\boldsymbol{R_1}$

$\boldsymbol{\xi_2}$ = orientation angle of $\boldsymbol{R_2}$

$\boldsymbol{\xi_3}$ = orientation angle of $\boldsymbol{R_3}$

R_1 = magnitude of $\boldsymbol{R_1}$

R_2 = magnitude of $\boldsymbol{R_2}$

R_3 = magnitude of $\boldsymbol{R_3}$

1st Dyade: WZ – Solution

Write the Vector loop of each precision position

$$W_1 + Z_1 = R_1$$

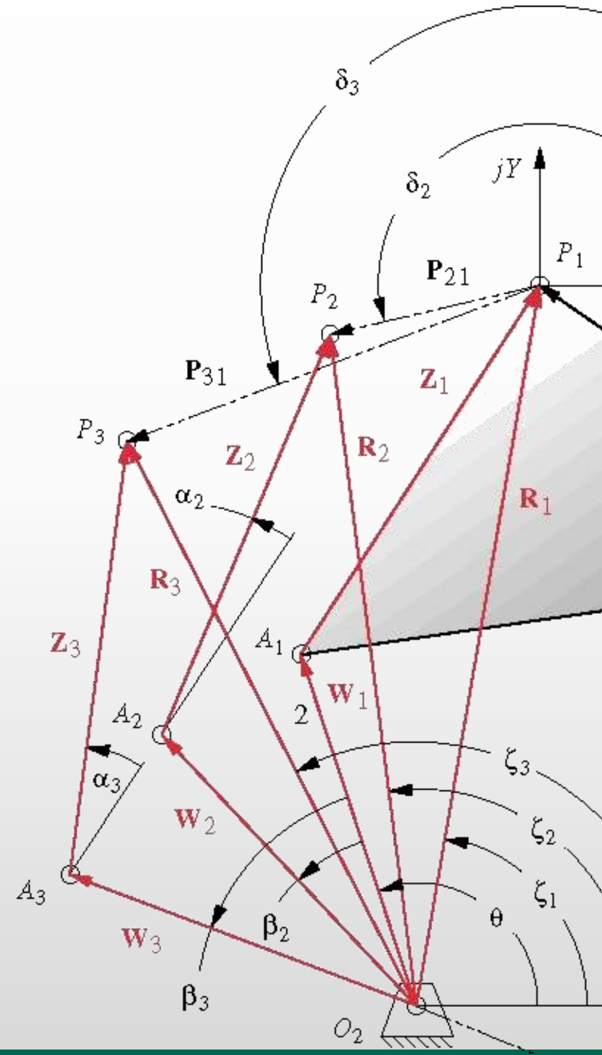
$$W_2 + Z_2 = R_2$$

$$W_3 + Z_3 = R_3$$

Substitute

$$W_1 = we^{j\theta}, W_2 = we^{j(\theta+\beta_2)} = W_1 e^{j\beta_2}$$

$$Z_1 = ze^{j\phi}, Z_2 = ze^{j(\phi+\alpha_2)} = Z_1 e^{j\alpha_2}$$



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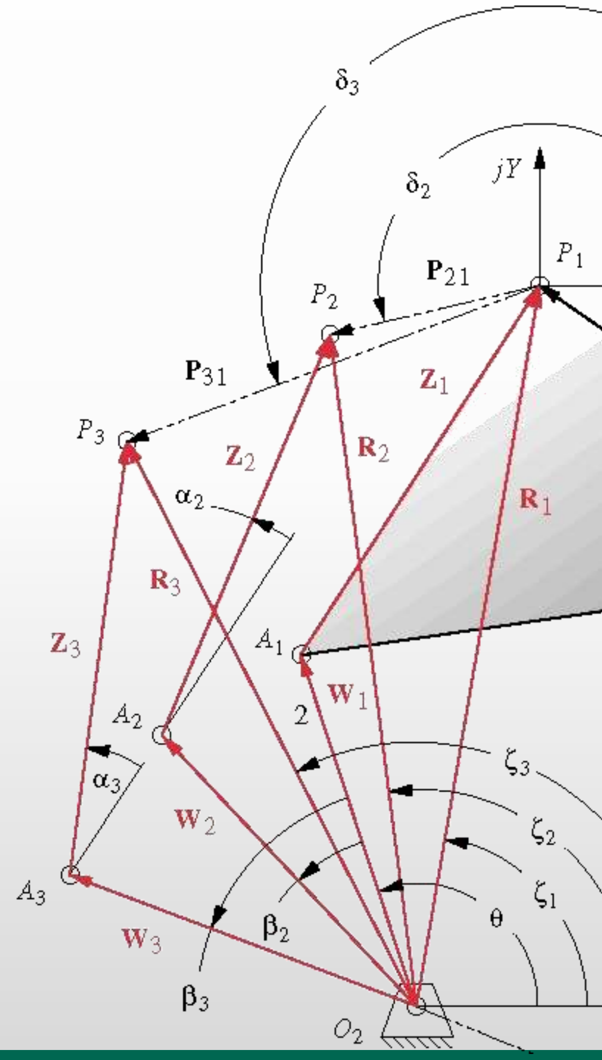
1st Dyade: WZ – Solution

Resultant system of equations

$$W_1 + Z_1 = R_1$$
$$W_1 e^{j\beta_2} + Z_1 e^{j\alpha_2} = R_2$$
$$W_1 e^{j\beta_3} + Z_1 e^{j\alpha_3} = R_3$$

Solution: determinant of the below matrix is zero

$$\begin{bmatrix} 1 & 1 & R_1 \\ e^{j\beta_2} & e^{j\alpha_2} & R_2 \\ e^{j\beta_3} & e^{j\alpha_3} & R_3 \end{bmatrix}$$



1st Dyade: WZ – Solution

The determinant can be simplified to

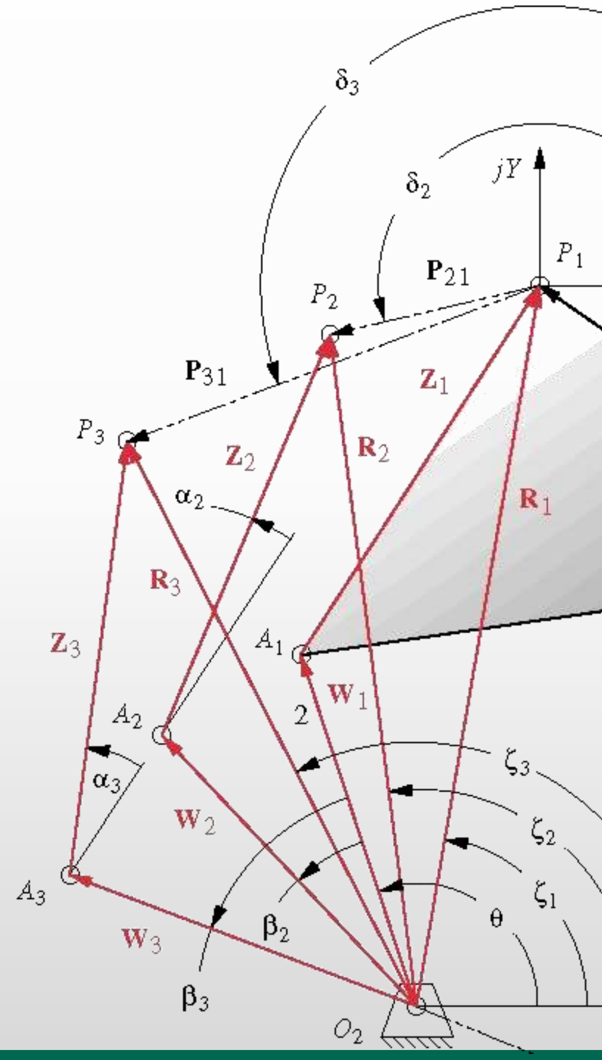
$$A + Be^{j\beta_2} + Ce^{j\beta_3} = 0$$

Where

$$A = R_3 e^{j\alpha_2} - R_2 e^{j\alpha_3}$$

$$B = R_1 e^{j\alpha_3} - R_3$$

$$C = R_2 \quad - R_1 e^{j\alpha_2}$$



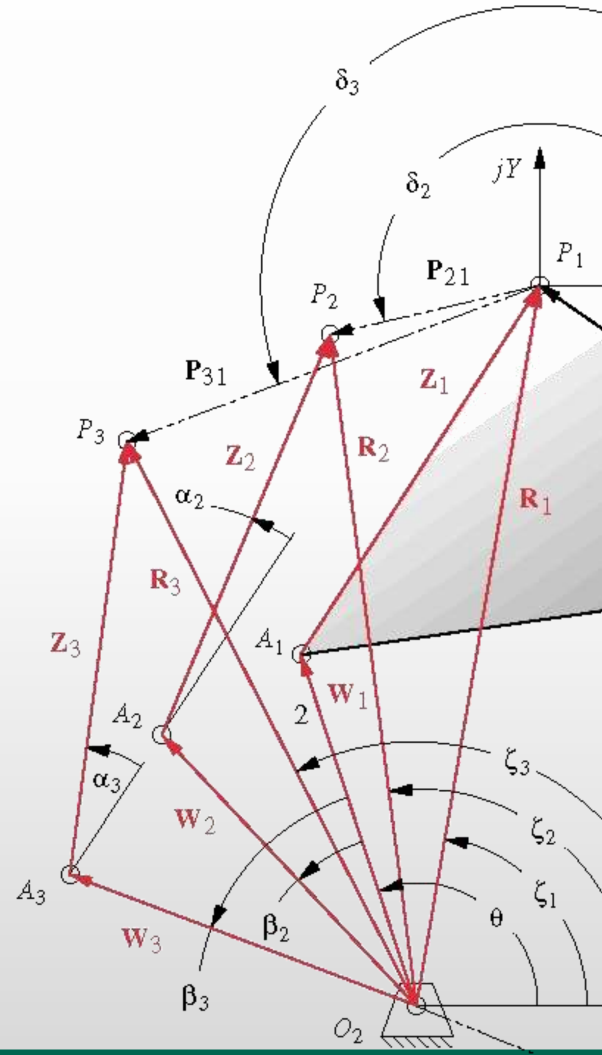
1st Dyade: WZ – Solution

Solving the real and imaginary part of the equation we get

$$\beta_3 = 2\arctan\left(\frac{K_2 \pm \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$

$$\beta_2 = \arctan\left[\frac{-\left(A_3 \sin\beta_3 + A_2 \cos\beta_3 + A_4\right)}{-\left(A_5 \sin\beta_3 + A_3 \cos\beta_3 + A_6\right)}\right]$$

Ignore the solution where $\beta_2 = \alpha_2 \; and \; \beta_3 = \alpha_3$



1st Dyade: WZ – Solution Constants

$$K_1 = A_2 A_4 + A_3 A_6$$

$$K_2 = A_3 A_4 + A_5 A_6$$

$$K_3 = \frac{\left(A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2\right)}{2}$$

$$A_1 = -C_3^2 - C_4^2 \qquad A_2 = C_3 C_6 - C_4 C_5$$

$$A_3 = -C_4 C_6 - C_3 C_5 \qquad A_4 = C_2 C_3 + C_1 C_4$$

$$A_5 = C_4 C_5 - C_3 C_6 \qquad A_6 = C_1 C_3 - C_2 C_4$$

$$C_1 = R_3 \cos(\alpha_2 + \zeta_3) - R_2 \cos(\alpha_3 + \zeta_2)$$

$$C_2 = R_3 \sin(\alpha_2 + \zeta_3) - R_2 \sin(\alpha_3 + \zeta_2)$$

$$C_3 = R_1 \cos(\alpha_3 + \zeta_1) - R_3 \cos\zeta_3$$

$$C_4 = -R_1 \sin(\alpha_3 + \zeta_1) + R_3 \sin\zeta_3$$

$$C_5 = R_1 \cos(\alpha_2 + \zeta_1) - R_2 \cos\zeta_2$$

$$C_6 = -R_1 \sin(\alpha_2 + \zeta_1) + R_2 \sin\zeta_2$$

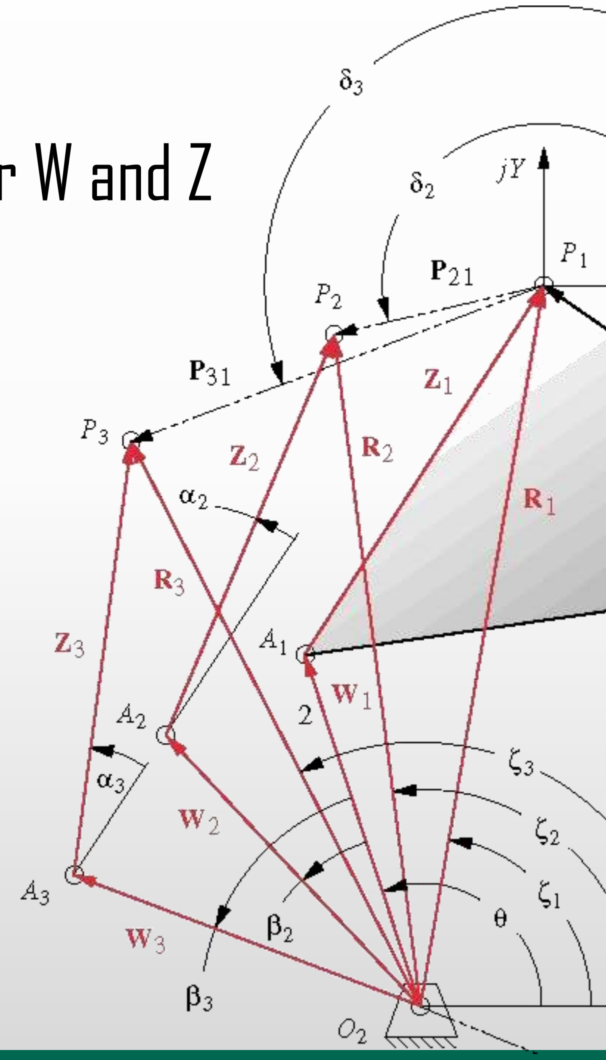
1st Dyade: WZ

Use the following vector loop equation to solve for W and Z

$$W_2 + Z_2 - P_{21} - Z_1 - W_1 = 0$$
$$W_3 + Z_3 - P_{31} - Z_1 - W_1 = 0$$

Unknowns

$$W_{1_x} = w\cos\theta \qquad Z_{1_x} = z\cos\phi$$
$$W_{1_y} = w\sin\theta \qquad Z_{1_y} = z\sin\phi$$



1st Dyade: WZ

Take the real and imaginary components of both equations

$$A W_{1_x} - B W_{1_y} + C Z_{1_x} - D Z_{1_y} = E$$

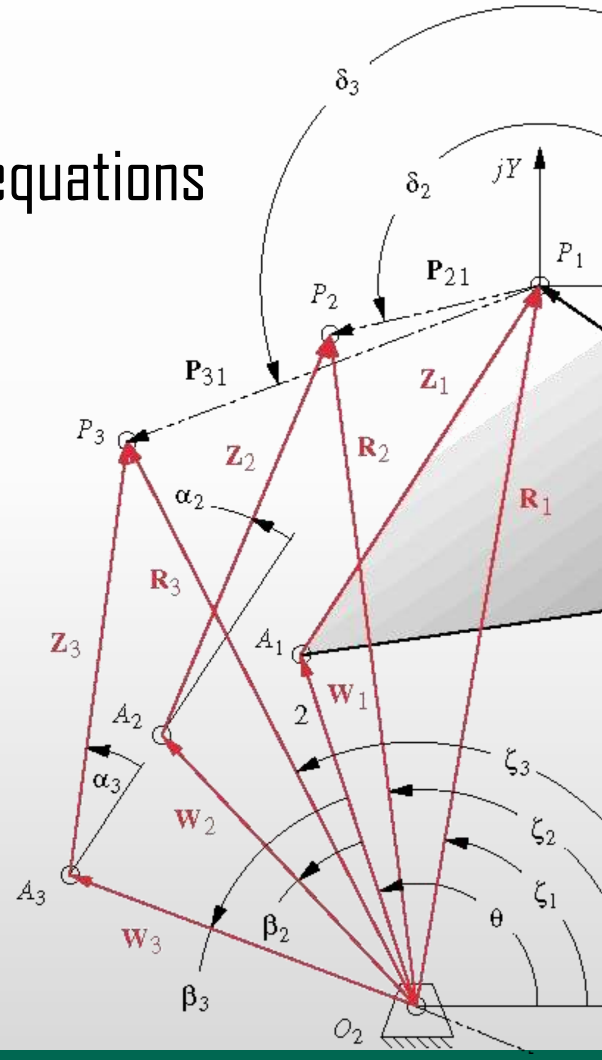
$$F W_{1_x} - G W_{1_y} + H Z_{1_x} - K Z_{1_y} = L$$

$$B W_{1_x} + A W_{1_y} + D Z_{1_x} + C Z_{1_y} = M$$

$$G W_{1_x} + F W_{1_y} + K Z_{1_x} + H Z_{1_y} = N$$

Where

$A = \cos\beta_2 - 1$	$B = \sin\beta_2$	$C = \cos\alpha_2 - 1$
$D = \sin\alpha_2$	$E = p_{21}\cos\delta_2$	$F = \cos\beta_3 - 1$
$G = \sin\beta_3$	$H = \cos\alpha_3 - 1$	$K = \sin\alpha_3$
$L = p_{31}\cos\delta_3$	$M = p_{21}\sin\delta_2$	$N = p_{31}\sin\delta_3$



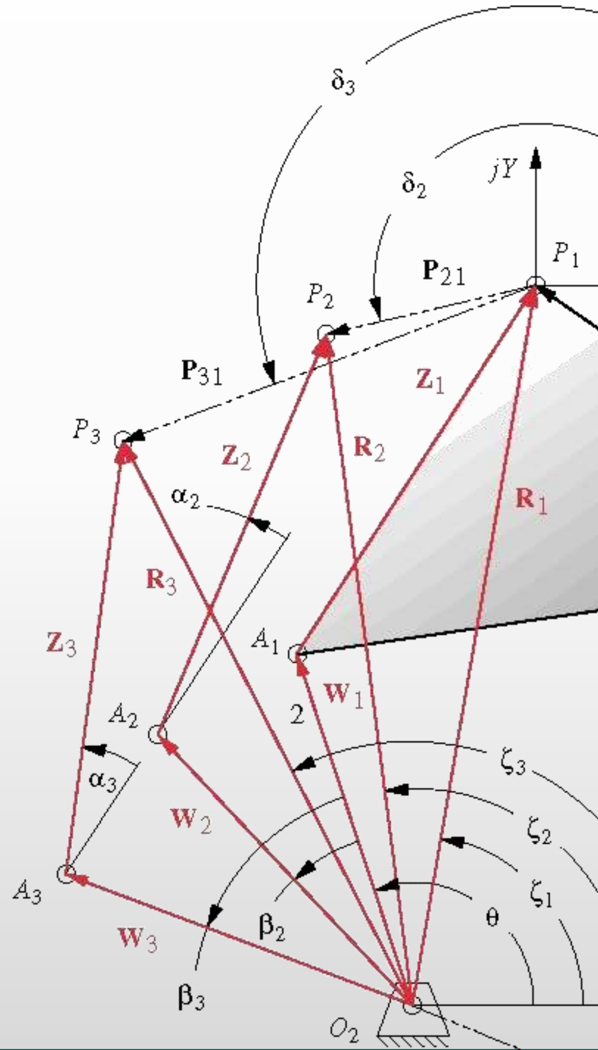
1st Dyade: WZ

Solve the system of equations using matrices

$$\begin{bmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{bmatrix} \times \begin{bmatrix} W_{1_x} \\ W_{1_y} \\ Z_{1_x} \\ Z_{1_y} \end{bmatrix} = \begin{bmatrix} E \\ L \\ M \\ N \end{bmatrix}$$

The matrix solution will give the solution to vectors W and Z.

Redo the same procedure for vector U and S



Solution Summary

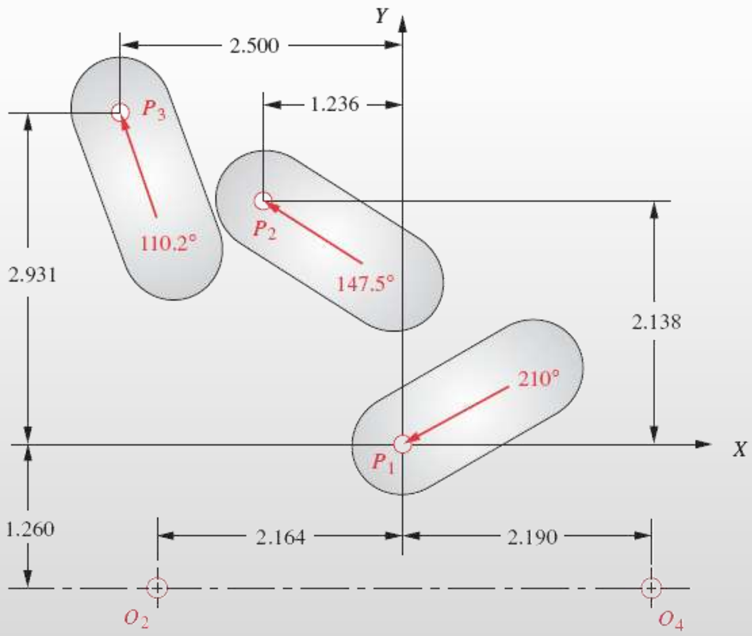
$$\beta_3 = 2\arctan\left(\frac{K_2 \pm \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$

$$\beta_2 = \arctan\left[\frac{-\left(A_3 \sin\beta_3 + A_2 \cos\beta_3 + A_4\right)}{-\left(A_5 \sin\beta_3 + A_3 \cos\beta_3 + A_6\right)}\right]$$

$$\begin{bmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{bmatrix} \times \begin{bmatrix} W_{1_x} \\ W_{1_y} \\ Z_{1_x} \\ Z_{1_y} \end{bmatrix} = \begin{bmatrix} E \\ L \\ M \\ N \end{bmatrix}$$

Example: Problem: 5-11 p.255

Design a linkage to carry the body in the figure below through the three positions P1, P2 and P3 at the angles shown in the figure. Use analytical synthesis and design it for the fixed pivots shown.

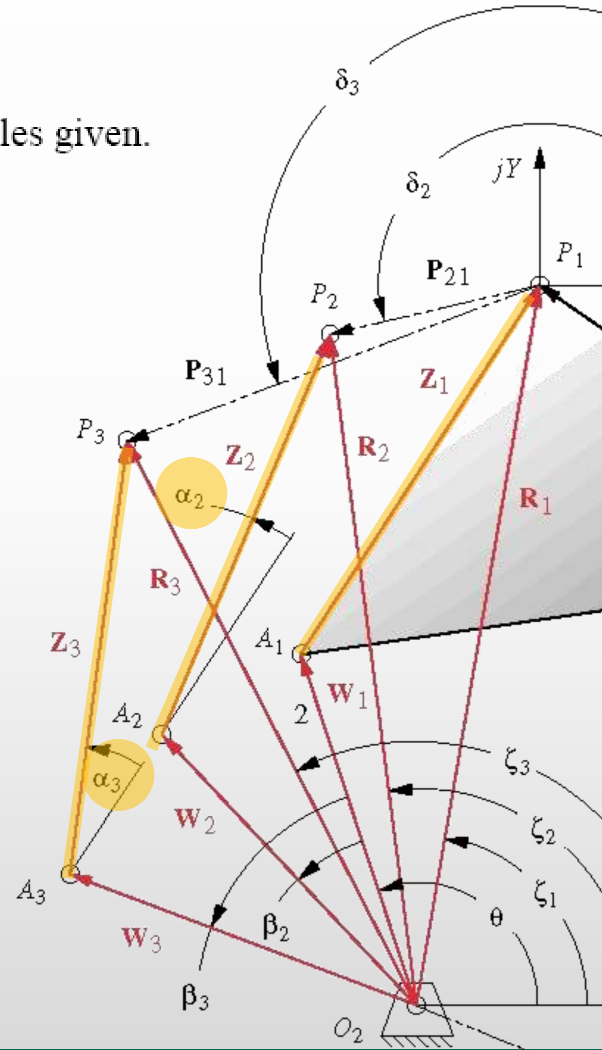


Example Solution – Step 1

Determine the angle changes between precision points from the body angles given.

$$\alpha_2 := \theta_{P2} - \theta_{P1} \qquad \alpha_2 = -62.500\ deg$$

$$\alpha_3 := \theta_{P3} - \theta_{P1} \qquad \alpha_3 = -99.800\ deg$$



Example Solution – Step 2

determine the magnitudes of $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ and their x and y components.

$R_{1x} := -O_{2x}$ $\quad R_{1x} = 2.164$ $\quad R_{1y} := -O_{2y}$ $\quad R_{1y} = 1.260$

$R_{2x} := R_{1x} + P_{21x}$ $\quad R_{2x} = 0.928$

$R_{2y} := R_{1y} + P_{21y}$ $\quad R_{2y} = 3.398$

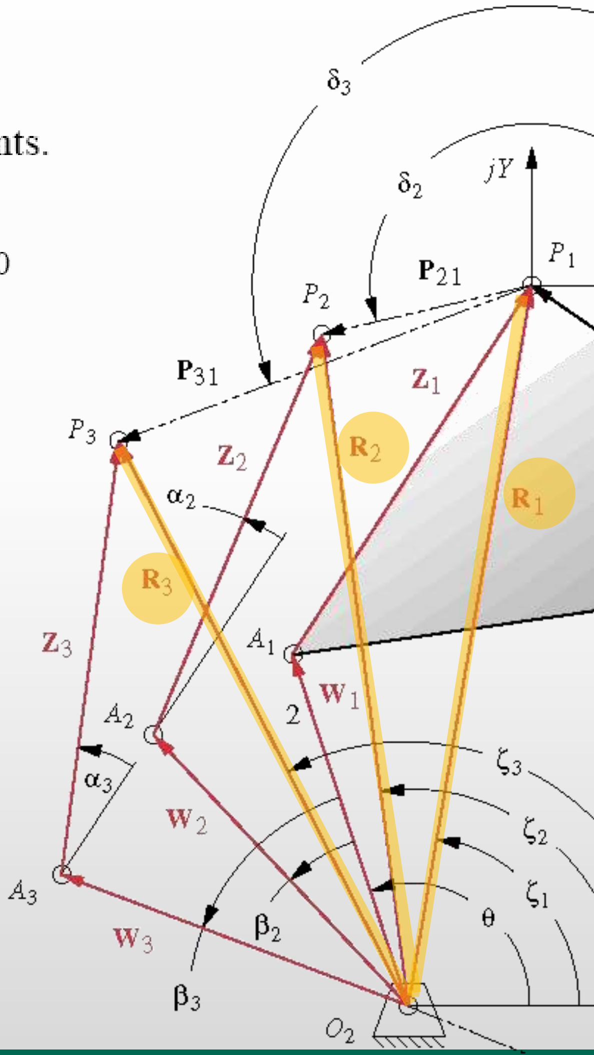
$R_{3x} := R_{1x} + P_{31x}$ $\quad R_{3x} = -0.336$

$R_{3y} := R_{1y} + P_{31y}$ $\quad R_{3y} = 4.191$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\quad R_1 = 2.504$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\quad R_2 = 3.522$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\quad R_3 = 4.204$



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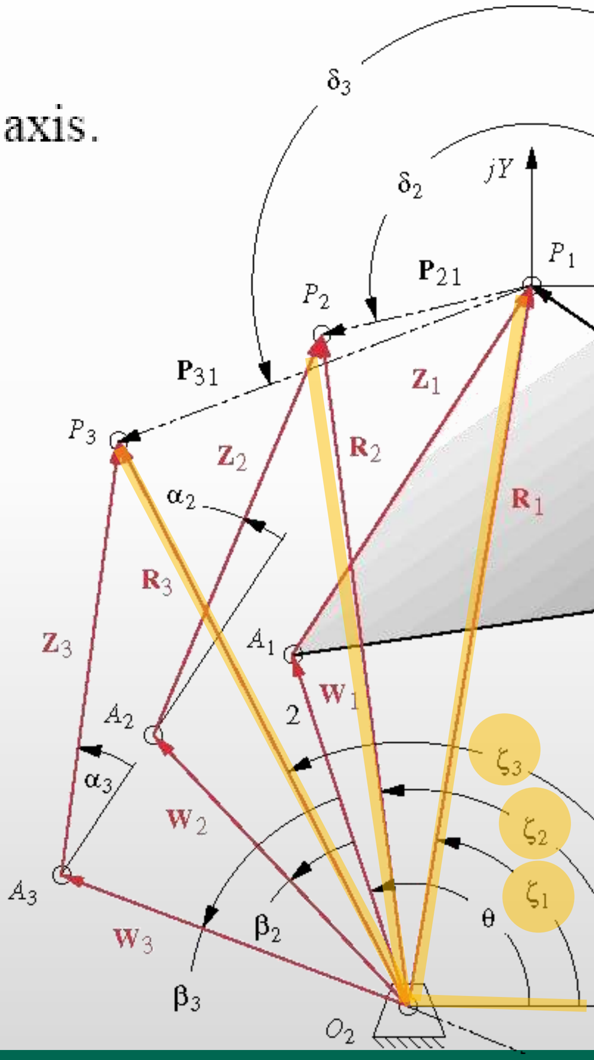
Example Solution – Step 3

determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := atan2(R_{1x}, R_{1y}) \qquad \zeta_1 = 30.210\,deg$$

$$\zeta_2 := atan2(R_{2x}, R_{2y}) \qquad \zeta_2 = 74.725\,deg$$

$$\zeta_3 := atan2(R_{3x}, R_{3y}) \qquad \zeta_3 = 94.584\,deg$$



Example Solution – Step 4a

Solve for β_2 and β_3

$$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2) \qquad C_1 = 0.372$$

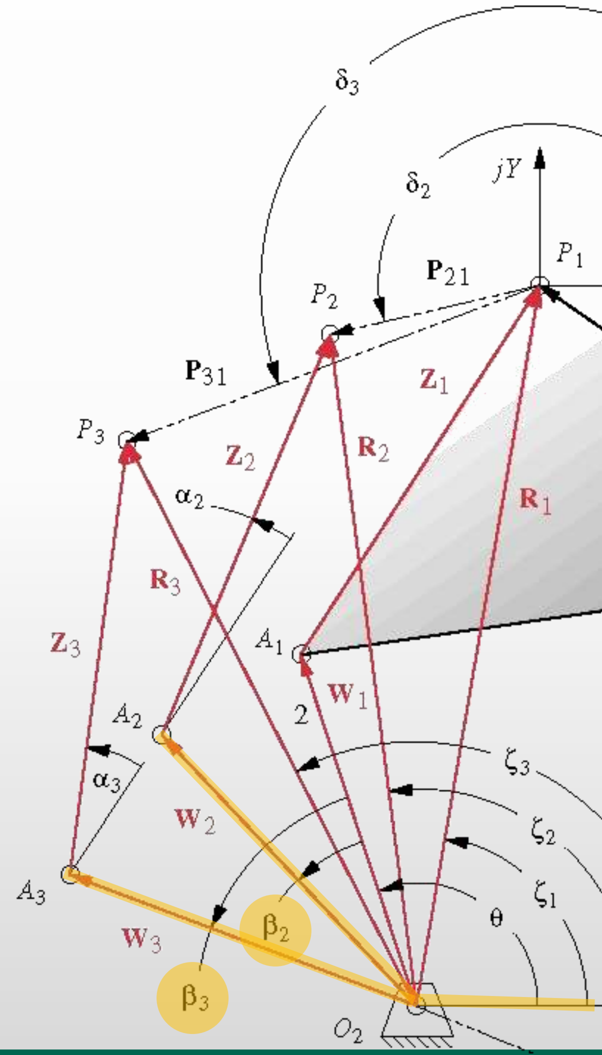
$$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2) \qquad C_2 = 3.726$$

$$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3) \qquad C_3 = 1.209$$

$$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3) \qquad C_4 = 6.538$$

$$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2) \qquad C_5 = 1.189$$

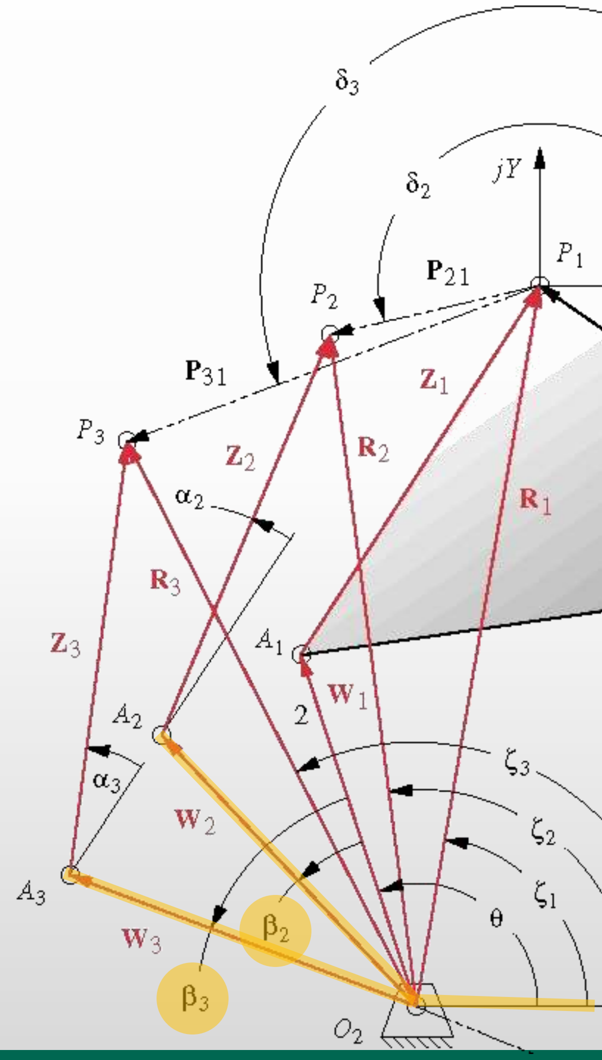
$$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2) \qquad C_6 = 4.736$$



Example Solution – Step 4b

Solve for β_2 and β_3

$A_1 := -C_3^{\,2} - C_4^{\,2}$	$A_1 = -44.206$
$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$	$A_2 = -2.046$
$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$	$A_3 = -32.399$
$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$	$A_4 = 6.937$
$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$	$A_5 = 2.046$
$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$	$A_6 = -23.911$
$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$	$K_1 = 760.497$
$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$	$K_2 = -273.669$
$K_3 := \dfrac{A_1^{\,2} - A_2^{\,2} - A_3^{\,2} - A_4^{\,2} - A_6^{\,2}}{2}$	$K_3 = 140.232$



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Example Solution – Step 4c

Solve for β_2 and β_3

$$\beta_{31} := 2\cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$ $\beta_{31} = 60.217\,deg$

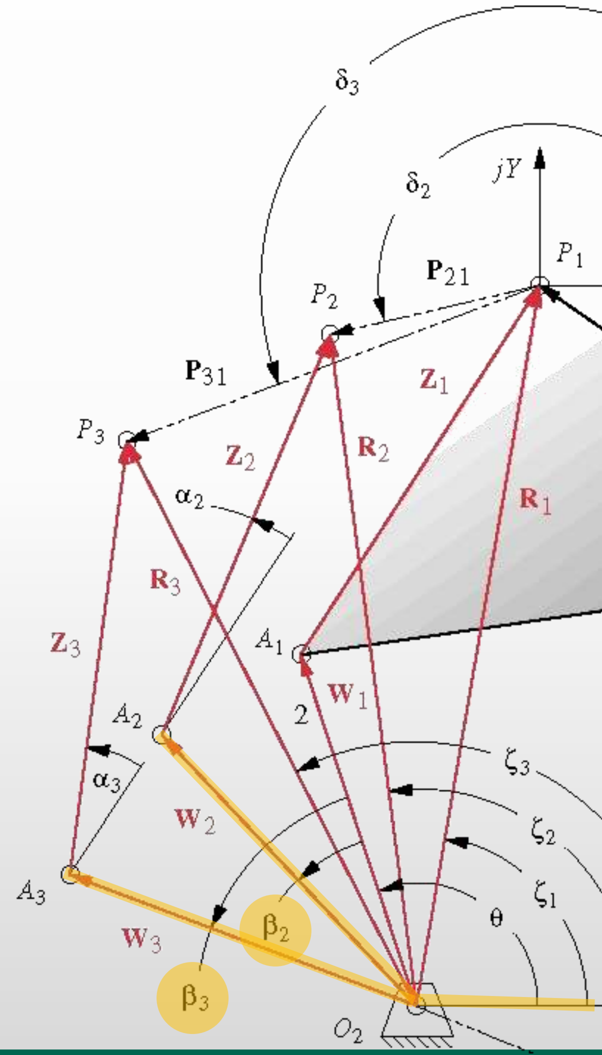
$$\beta_{32} := 2\cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right)$$ $\beta_{32} = -99.800\,deg$

The second value is the same as α_3, so use the first value $\beta_3 := \beta_{31}$

$$\beta_{21} := acos\left(\frac{A_5\cdot sin(\beta_3) + A_3\cdot cos(\beta_3) + A_6}{A_1}\right)$$ $\beta_{21} = 30.143\,deg$

$$\beta_{22} := asin\left(\frac{A_3\cdot sin(\beta_3) + A_2\cdot cos(\beta_3) + A_4}{A_1}\right)$$ $\beta_{22} = 30.143\,deg$

Since both values are the same, $\beta_2 := \beta_{21}$



Example Solution – Step 5

Repeat steps 2, 3, and 4 for the right-hand dyad to find γ_1 and γ_2.

$R_{1x} := -O_{4x}$ $\qquad R_{1x} = -2.190$

$R_{1y} := -O_{4y}$ $\qquad R_{1y} = 1.260$

$R_{2x} := R_{1x} + P_{21x}$ $\qquad R_{2x} = -3.426$

$R_{2y} := R_{1y} + P_{21y}$ $\qquad R_{2y} = 3.398$

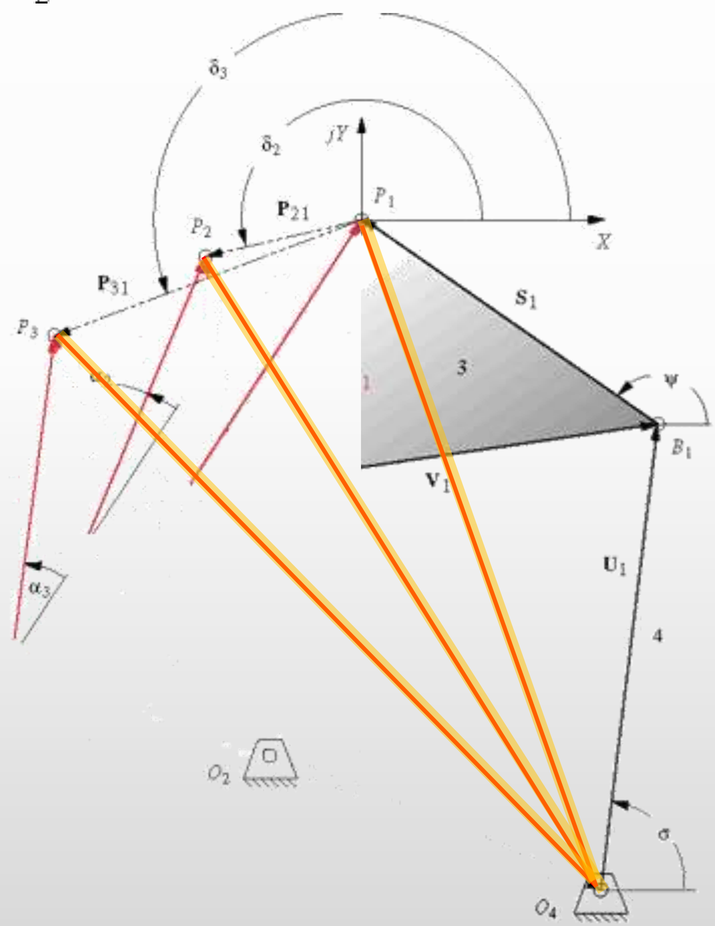
$R_{3x} := R_{1x} + P_{31x}$ $\qquad R_{3x} = -4.690$

$R_{3y} := R_{1y} + P_{31y}$ $\qquad R_{3y} = 4.191$

$R_1 := \sqrt{R_{1x}^2 + R_{1y}^2}$ $\qquad R_1 = 2.527$

$R_2 := \sqrt{R_{2x}^2 + R_{2y}^2}$ $\qquad R_2 = 4.825$

$R_3 := \sqrt{R_{3x}^2 + R_{3y}^2}$ $\qquad R_3 = 6.290$



Example Solution – Step 6

determine the angles that $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ make with the x axis.

$$\zeta_1 := atan2\left(R_{1x}, R_{1y}\right)$$

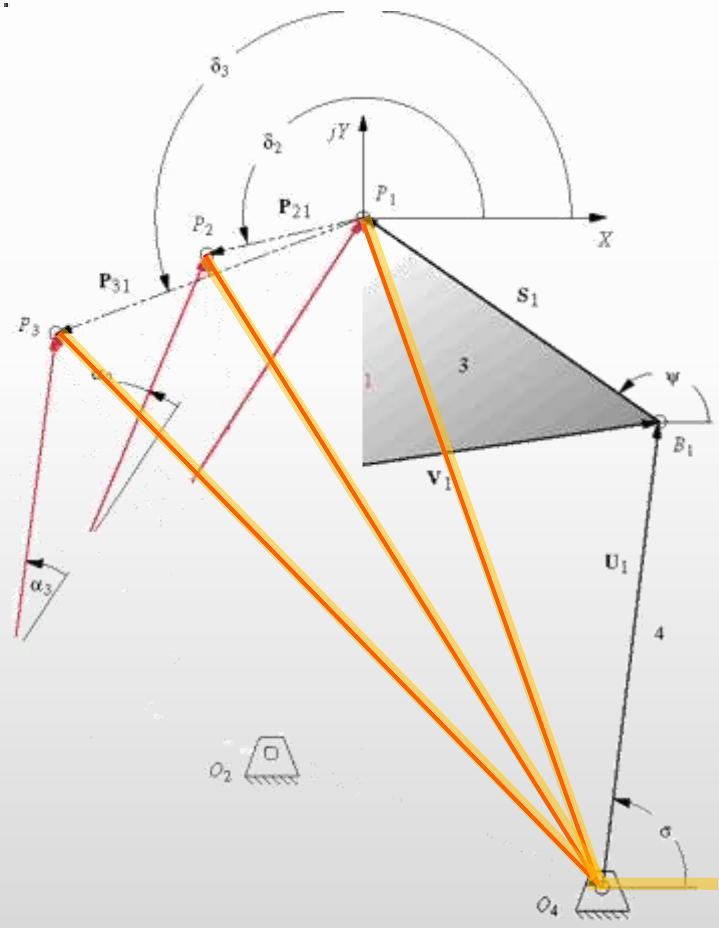
$$\zeta_2 := atan2\left(R_{2x}, R_{2y}\right)$$

$$\zeta_3 := atan2\left(R_{3x}, R_{3y}\right)$$

$$\zeta_1 = 150.086\,deg$$

$$\zeta_2 = 135.235\,deg$$

$$\zeta_3 = 138.216\,deg$$



Example Solution – Step 7a

Solve for γ_2 and γ_3

$C_1 := R_3 \cdot cos(\alpha_2 + \zeta_3) - R_2 \cdot cos(\alpha_3 + \zeta_2)$	$C_1 = -2.380$
$C_2 := R_3 \cdot sin(\alpha_2 + \zeta_3) - R_2 \cdot sin(\alpha_3 + \zeta_2)$	$C_2 = 3.298$
$C_3 := R_1 \cdot cos(\alpha_3 + \zeta_1) - R_3 \cdot cos(\zeta_3)$	$C_3 = 6.304$
$C_4 := -R_1 \cdot sin(\alpha_3 + \zeta_1) + R_3 \cdot sin(\zeta_3)$	$C_4 = 2.247$
$C_5 := R_1 \cdot cos(\alpha_2 + \zeta_1) - R_2 \cdot cos(\zeta_2)$	$C_5 = 3.532$
$C_6 := -R_1 \cdot sin(\alpha_2 + \zeta_1) + R_2 \cdot sin(\zeta_2)$	$C_6 = 0.874$

Example Solution – Step 7b

$A_1 := -C_3^2 - C_4^2$ $\quad A_1 = -44.796$

$A_2 := C_3 \cdot C_6 - C_4 \cdot C_5$ $\quad A_2 = -2.431$

$A_3 := -C_4 \cdot C_6 - C_3 \cdot C_5$ $\quad A_3 = -24.233$

$A_4 := C_2 \cdot C_3 + C_1 \cdot C_4$ $\quad A_4 = 15.441$

$A_5 := C_4 \cdot C_5 - C_3 \cdot C_6$ $\quad A_5 = 2.431$

$A_6 := C_1 \cdot C_3 - C_2 \cdot C_4$ $\quad A_6 = -22.414$

$K_1 := A_2 \cdot A_4 + A_3 \cdot A_6$ $\quad K_1 = 505.612$

$K_2 := A_3 \cdot A_4 + A_5 \cdot A_6$ $\quad K_2 = -428.679$

$$K_3 := \frac{A_1^2 - A_2^2 - A_3^2 - A_4^2 - A_6^2}{2}$$ $\quad K_3 = 336.363$

Example Solution – Step 7c

$$\gamma_{31} := 2 \cdot atan\left(\frac{K_2 + \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{31} = 19.215\,deg$$

$$\gamma_{32} := 2 \cdot atan\left(\frac{K_2 - \sqrt{K_1^2 + K_2^2 - K_3^2}}{K_1 + K_3}\right) \qquad \gamma_{32} = -99.800\,deg$$

The second value is the same as α_3, so use the first value $\qquad \gamma_3 := \gamma_{31}$

$$\gamma_{21} := acos\left(\frac{A_5 \cdot sin(\gamma_3) + A_3 \cdot cos(\gamma_3) + A_6}{A_1}\right) \qquad \gamma_{21} = 6.628\,deg$$

$$\gamma_{22} := asin\left(\frac{A_3 \cdot sin(\gamma_3) + A_2 \cdot cos(\gamma_3) + A_4}{A_1}\right) \qquad \gamma_{22} = -6.628\,deg$$

Since γ_2 is not in the first quadrant , $\qquad \gamma_2 := \gamma_{22}$

Example Solution – Step 8

Solve for the linkage vectors as described on slide 11.

Start by finding the magnitudes of vectors P_{21} and P_{31} and its angles:

$$p_{21} := \sqrt{{P_{21x}}^2 + {P_{21y}}^2} \qquad p_{21} = 2.470$$

$$\delta_2 := atan2(P_{21x}, P_{21y}) \qquad \delta_2 = 120.033\ deg$$

$$p_{31} := \sqrt{{P_{31x}}^2 + {P_{31y}}^2} \qquad p_{31} = 3.852$$

$$\delta_3 := atan2(P_{31x}, P_{31y}) \qquad \delta_3 = 130.463\ deg$$

Example Solution – Step 9

Evaluate terms in the WZ coefficient matrix

$A := \cos(\beta_2) - 1$ $B := \sin(\beta_2)$ $C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $E := p_{21} \cdot \cos(\delta_2)$ $F := \cos(\beta_3) - 1$

$G := \sin(\beta_3)$ $H := \cos(\alpha_3) - 1$ $K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $M := p_{21} \cdot \sin(\delta_2)$ $N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A & -B & C & -D \\ F & -G & H & -K \\ B & A & D & C \\ G & F & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} W1x \\ W1y \\ Z1x \\ Z1y \end{pmatrix} := AA^{-1} \cdot CC$$

Example Solution – Step 10

The components of the W and Z vectors are

$W1x = 2.915$ $W1y = 1.702$ $Z1x = -0.751$ $Z1y = -0.442$

And the length of link 2 is

$$w := \sqrt{W1x^2 + W1y^2} \qquad w = 3.376$$

Example Solution – Step 11

Evaluate terms in the US coefficient matrix

$A' := \cos(\gamma_2) - 1$ $\quad B' := \sin(\gamma_2)$ $\quad C := \cos(\alpha_2) - 1$

$D := \sin(\alpha_2)$ $\quad E := p_{21} \cdot \cos(\delta_2)$ $\quad F' := \cos(\gamma_3) - 1$

$G' := \sin(\gamma_3)$ $\quad H := \cos(\alpha_3) - 1$ $\quad K := \sin(\alpha_3)$

$L := p_{31} \cdot \cos(\delta_3)$ $\quad M := p_{21} \cdot \sin(\delta_2)$ $\quad N := p_{31} \cdot \sin(\delta_3)$

$$AA := \begin{pmatrix} A' & -B' & C & -D \\ F' & -G' & H & -K \\ B' & A' & D & C \\ G' & F' & K & H \end{pmatrix} \qquad CC := \begin{pmatrix} E \\ L \\ M \\ N \end{pmatrix} \qquad \begin{pmatrix} U1x \\ U1y \\ S1x \\ S1y \end{pmatrix} := AA^{-1} \cdot CC$$

Example Solution – Step 12

The components of the U and S vectors are

$U1x = -1.371$ $U1y = 3.634$ $S1x = -0.819$ $S1y = -2.374$

And the length of link 4 is

$u := \sqrt{U1x^2 + U1y^2}$ $u = 3.884$

Example Solution – Step 13

Solving for links 3 and 1

$V1x := Z1x - S1x$ $\quad V1x = 0.068$

$V1y := Z1y - S1y$ $\quad V1y = 1.932$

The length of link 3 is: $v := \sqrt{V1x^2 + V1y^2}$ $\quad v = 1.933$

$G1x := W1x + V1x - U1x$ $\quad G1x = 4.354$

$G1y := W1y + V1y - U1y$ $\quad G1y = -4.441 \times 10^{-15}$

The length of link 1 is: $g := \sqrt{G1x^2 + G1y^2}$ $\quad g = 4.354$

Example Solution – Step 14

Check the location of the fixed pivot points with respect to the global frame using the calculated vectors W1, Z1, U1, and S1

$O2x := -Z1x - W1x$	$O2x = -2.164$
$O2y := -Z1y - W1y$	$O2y = -1.260$
$O4x := -S1x - U1x$	$O4x = 2.190$
$O4y := -S1y - U1y$	$O4y = -1.260$

Example Solution – Step 15

Determine the location of the coupler point with respect to point A and line AB.

Distance from A to P $\quad z := \sqrt{Z1x^2 + Z1y^2}$ $\quad z = 0.871$ $\quad r_P := z$

Angle BAP (δ_p) $\quad s := \sqrt{S1x^2 + S1y^2}$ $\quad s = 2.511$

$\psi := atan2(S1x, S1y)$ $\quad \psi = 250.963\ deg$

$\phi := atan2(Z1x, Z1y)$ $\quad \phi = 210.445\ deg$

$\theta_3 := atan2(z \cdot cos(\phi) - s \cdot cos(\psi), z \cdot sin(\phi) - s \cdot sin(\psi))$

$\theta_3 = 87.994\ deg$

$\delta_p := \phi - \theta_3$ $\quad \delta_p = 122.451\ deg$

Example Solution – Summary

DESIGN SUMMARY

Link 1:	$g = 4.354$
Link 2:	$w = 3.376$
Link 3:	$v = 1.933$
Link 4:	$u = 3.884$
Coupler point:	$r_P = 0.871$ $\delta_p = 122.451\ deg$

VERIFICATION: The calculated values of g (length of the ground link) and of the coordinates of O_2 and O_4 give the same values as those on the problem statement, verifying that the calculated values for the other links and the coupler point are correct.